**4E-2** Find the rectangular equation for $x = t + 1/t$ and $y = t - 1/t$ (compute $x^2$ and $y^2$).

**4E-3** Find the rectangular equation for $x = 1 + \sin t$, $y = 4 + \cos t$.

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2. $x = t + \frac{1}{t}$ , $y = t - \frac{1}{t}$

$x^2 = t^2 + 2 + \frac{1}{t^2}$

$y^2 = t^2 - 2 + \frac{1}{t^2}$

$x^2 - y^2 = 4$

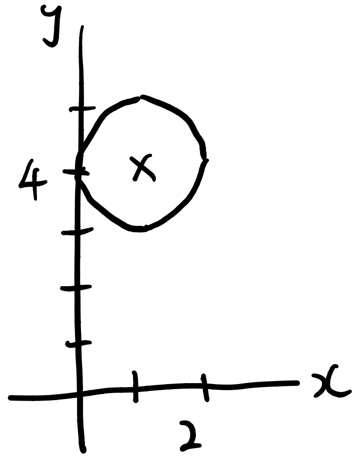
$y^2 = x^2 - 4$

3. $x = 1 + \sin t$ , $y = 4 + \cos t$

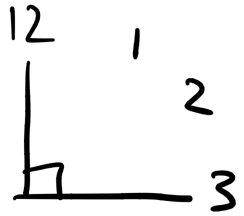
$\sin t = x - 1$ , $\cos t = y - 4$

$\Rightarrow \sin^2 t + \cos^2 t = (x-1)^2 + (y-4)^2$

$(x-1)^2 + (y-4)^4 = 1$



**4E-8** At noon, a snail starts at the center of an open clock face. It creeps at a steady rate along the hour hand, reaching the end of the hand at 1:00 PM. The hour hand is 1 meter long. Write parametric equations for the position of the snail at time t, in some reasonable xy-coordinate system.


$x(0) = 0$

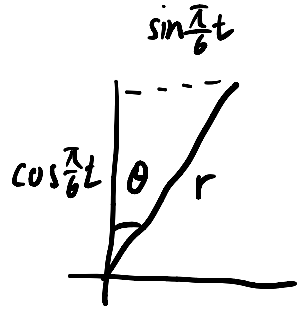
$y(0) = 0$

$t$ : time in hours.

$x(t) = \sin \frac{\pi}{6} t$

$y(t) = \cos \frac{\pi}{6} t$

$$\frac{\frac{\pi}{2}}{3} = \frac{\pi}{6}$$




$r^2 = x^2 + y^2$

$= k^2 \sin^2 \frac{\pi}{6} t + k^2 \cos^2 \frac{\pi}{6} t$

$= k^2$

$r = k$

At $t = 0$, $k = 0$,

at $t = 1$, $k = 1$

$\therefore r = t$

$\therefore (x, y) = \left(t \sin \frac{\pi}{6} t,\ t \cos \frac{\pi}{6} t\right)$

**4F-1** Find the arclength of the following curves

a) $y = 5x + 2$, $0 \le x \le 1$.
b) $y = x^{3/2}$, $0 \le x \le 1$.
c) $y = (1 - x^{2/3})^{3/2}$, $0 \le x \le 1$.
~~d)~~ $y = (1/3)(2 + x^2)^{3/2}$, $1 \le x \le 2$.

d) $y = \frac{1}{3}(2+x^2)^{3/2}$, $1 \le x \le 2$

$$\frac{dy}{dx} = \frac{1}{2}(2+x^2)^{\frac{1}{2}}(2x)$$
$$= x\sqrt{2+x^2}$$

$$ds = \sqrt{dx^2 + dy^2}$$

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\int \frac{ds}{dx}\,dx = \int_1^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$s = \int_1^2 \sqrt{1+x^2(2+x^2)}\,dx$$

$$= \int_1^2 \sqrt{1+2x^2+x^4}\,dx$$

$$= \int_1^2 \sqrt{(x^2+1)^2}\,dx$$

$$= \int_1^2 x^2+1\,dx$$

$$= \frac{x^3}{3} + x \Big|_1^2$$

$$= \left(\frac{8}{3}+2\right) - \left(\frac{1}{3}+1\right)$$

$$= \frac{10}{3}$$

**4F-4** Find the length of the curve $x = t^2$, $y = t^3$ for $0 \le t \le 2$.

**4F-5** Find an integral for the length of the curve given parametrically in Exercise 4E-2 for $1 \le t \le 2$. Simplify the integrand as much as possible but do not evaluate.

4. $x = t^2, \quad y = t^3 \qquad 0 \le t \le 2$

$$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 3t^2$$

$$ds^2 = dx^2 + dy^2$$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$u = 4 + 9t^2 \Rightarrow du = 18t\, dt$$

$$t = 0 \Rightarrow u = 4$$

$$t = 2 \Rightarrow u = 40$$

$$\frac{ds}{dt} = \sqrt{4t^2 + 9t^4}$$

$$\int \frac{ds}{dt}\, dt = \int_0^2 t\sqrt{4 + 9t^2}\, dt$$

$$s = \frac{1}{18}\int_4^{40} \sqrt{u}\, du$$

$$= \frac{1}{18} \frac{u^{3/2}}{3/2}\Bigg|_4^{40}$$

$$= \frac{1}{27}\left(80\sqrt{10} - 8\right)$$

5. $x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}$

$$\frac{dx}{dt} = 1 - \frac{1}{t^2}, \quad \frac{dy}{dt} = 1 + \frac{1}{t^2}$$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\int \frac{ds}{dt}\, dt = \int_1^2 \sqrt{1 - \frac{2}{t^2} + \frac{1}{t^4} + 1 + \frac{2}{t^2} + \frac{1}{t^4}}\, dt$$

$$s = \int_1^2 \sqrt{2 + \frac{2}{t^4}}\, dt$$

$$= \int_1^2 \frac{\sqrt{2t^4 + 2}}{t^2}\, dt$$

**4F-8** Find the length of the curve $x = e^t \cos t$, $y = e^t \sin t$ for $0 \le t \le 10$.

$x = e^t \cos t, \quad y = e^t \sin t \qquad 0 \le t \le 10$

$$\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$$

$$S = \int_0^{10} \sqrt{e^{2t}(1 - 2\sin t\cos t + 1 + 2\sin t \cos t)}\, dt$$

$$= \int_0^{10} e^t \sqrt{2}\, dt$$

$$= \sqrt{2} e^t \Big|_0^{10}$$

$$= \sqrt{2}(e^{10} - 1)$$

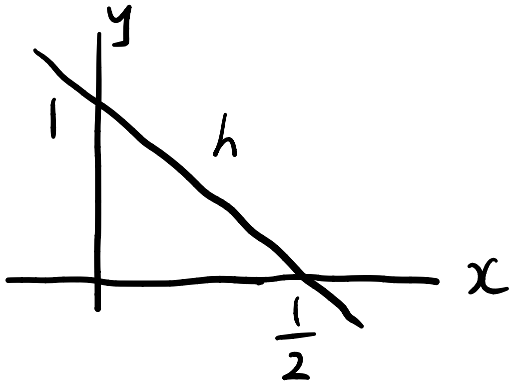
$$\frac{dx}{dt} = e^t \cos t + e^t(-\sin t)$$
$$= e^t(\cos t - \sin t)$$

$$\frac{dy}{dt} = e^t \sin t + e^t \cos t$$
$$= e^t(\sin t + \cos t)$$

$$\left(\frac{dx}{dt}\right)^2 = e^{2t}(\cos^2 t - 2\sin t\cos t + \sin^2 t)$$
$$= e^{2t}(1 - 2\sin t \cos t)$$

$$\left(\frac{dy}{dt}\right)^2 = e^{2t}(\sin^2 t + 2\sin t\cos t + \cos^2 t)$$
$$= e^{2t}(1 + 2\sin t\cos t)$$

**4G-2** Find the area of the segment of $y = 1 - 2x$ in the first quadrant revolved around the $x$-axis.




$$h = \sqrt{1^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$

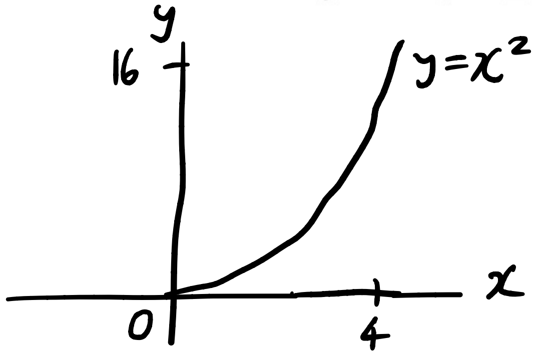
$$\frac{dy}{dx} = -2$$

$$dA = 2\pi y \, ds = 2\pi(1-2x)\sqrt{5}\,dx$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \sqrt{1+4}\,dx = \sqrt{5}\,dx$$

$$A = \int_0^{1/2} 2\sqrt{5}\pi(1-2x)\,dx = 2\sqrt{5}\pi\left(x - x^2\right)\Big|_0^{1/2} = 2\sqrt{5}\pi\left(\frac{1}{4}\right) = \frac{\sqrt{5}}{2}\pi$$

**4G-5** Find the area of $y = x^2$, $0 \le x \le 4$ revolved around the $y$-axis.




$$ds = \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}\ dy$$

$$= \sqrt{\frac{1}{4y} + 1}\ dy$$

$$dA = 2\pi x\ ds$$

$$= 2\pi\sqrt{y}\cdot\sqrt{\frac{1}{4y}+1}\,dy$$

$$A = \int_0^{16} 2\pi\sqrt{\frac{1}{4}+y}\ dy$$

$$= 2\pi\ \frac{\left(\frac{1}{4}+y\right)^{3/2}}{3/2}\Bigg|_0^{16}$$

$$= \frac{4}{3}\pi\left(\left(\frac{1}{4}+16\right)^{3/2} - \left(\frac{1}{4}\right)^{3/2}\right)$$

$$= \frac{4}{3}\pi\ \left(\frac{65}{4} - \frac{1}{4}\right)^{3/2}$$

$$= \frac{4}{3}\pi\cdot 64$$

$$= \frac{256}{3}\pi$$

$$y = x^2$$

$$\frac{d}{dy}y = \frac{d}{dy}x^2$$

$$1 = 2x\frac{dx}{dy}$$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{2x},\ \left(\frac{dx}{dy}\right)^2 = \frac{1}{4x^2} = \frac{1}{4y}$$